DP-003-2016001 Seat No. ________

B. Sc. (Sem. VI) (CBCS) (W.E.F. 2019) Examination

March - 2022

Graph Theory and Complex analysis-II : P-8(A)

Faculty Code : 003
Subject Code : 2016001

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Attempt any five questions out of ten.
(2) Right hand side figure indicate the marks.

1 (A) Answer the following questions in short : **4**

(1) A complete graph with 7 vertices has ________ edge-disjoint Hamiltonian circuits.

(2) Rank of a connected graph with n vertices and e edges is _______.

(3) Define isomorphic graphs.

(4) State the first theorem of graph theory.

(B) Attempt any one out of two : **2**

(1) If graph has exactly two vertices of odd degree, then prove that there must be a path joining these two vertices.

(2) In any tree (with 2 or more vertices). prove that there are at least two pendent vertices.

(C) Attempt any one of two : **3**

(1) Prove that in a binary tree with n vertices $\frac{n+1}{2}$ has pendent vertices.

(2) Prove that the number of odd vertices in a graph are always even.

(D) Attempt any one out of two : **5**

(1) If a graph G with n vertices is a tree, then prove that it has n-1 edges. Discuss its converse with agrument.

(2) Prove that a given connected graph G is Euler if and only if all the vertices of G are of even degree.

2 (A) Answer the following questions in short : **4**

(1) Write adjacency martrix of complete graph with 5 vertices.

(2) The chromatics number of binary tree with 11 vertices and of 5 level is __________.

(3) Define : acyclic graph.

(4) Define : properly colored graph.

(B) Attempt any one out of two : **2**

(1) In a standard notation fill in the blanks.

$n^* =$ _____, $f^* =$ _____, $r^* =$ ______, $\mu^* =$ _____.

(2) Prove that the reduced incidence matrix of a tree is non singular.

(C) Attempt any one out of two : **3**

(1) Prove that the ring sum of two circuits in a graph G is either a circuit or an edge-disjoint union of circuits.

(2) Show that every circuit has an even number of edges in common with any cut-set.

(D) Answer any one in detail : **5**

(1) With respect to a spanning tree T, prove that a branch b_i that determines a fundamental cut-set S is contained in verty fundamental circuit associated with the chords in S, and no other.

(2) Show that a connected plannar graph with n vertices and e edges has $e-n+2$ regions.

3 (A) Answr the following questions is short : **4**

(1) The point that coincides with the transformation is called ________.

(2) Define : mobius mapping.

(3) Define : conformal mapping.

(4) Define : bilinear map.

(B) Attempt any one out of two : **2**

(1) Find the fixed point of $w = \dfrac{z-1}{z+1}$.

(2) Show that $x+y=2$ transform into the parabola $u^2 = 8(v-2)$ under the transformation $w = z^2$.

(C) Attempt any one out of two : **3**

(1) Find bilinear transformation that maps $-1, 0$ and 1 into $-i$ and i respectively.

(2) Obtain a transformation of sector $r \leq 1, \leq \theta \leq \frac{\pi}{4}$ under the map $w = z^2$.

(D) Attempt any one out of two : **5**

(1) Prove that the transformation $(w+1)^2 = \frac{4}{z}$ transformation the unit circle of w-plane into parabola.

(2) Show that the composition of two bilinear transformations is again a bilinear transformation.

4 (A) Answer the following questions in short : **4**

(1) Find the radius of convergence for the series $\sum_{n=0}^{\infty} \frac{z^n}{n!}$

(2) Define : partial sum sequence for a complesx series.

(3) Write Maclaurian series expansion of sin z.

(4) State Laurent's theorem.

(B) Attemp any one out of two : **2**

(1) Find the limit of $\{z_1\}_{n=1}^{\infty}$, where $z_n = \frac{1}{n^3} + i.$

(2) If a series of complex numbers converges, then show that the nth term converges to zero as n tends to infinity.

(C) Attempt any one out of two : **3**

(1) Expand $\frac{z}{(z-2)(z+3)}$ in Laurent's series for $0 <| z-2 |< 1$.

(2) Show that z cos $z\, cohs\, z^2 \sum_{n=0}^{\infty} \frac{z^{4n+1}}{(2n)!}$

(D) Attempt any one out of two : **5**

(1) State and prove Taylor's theorem.

(2) Expand $f(z)=\frac{1}{z^2 \sinh z}$ in Laurent's series for $Z_0=0$ and hence deduce that $\int_C \frac{1}{\sinh z} dz = 2\pi i$ and $\int_C \frac{1}{z^2 \sinh z} dz = -\frac{\pi}{3} i$

5 (A) Answer the following questions in short : **4**

(1) $\operatorname{Re} s\left(\frac{\cos z}{z}, 0\right) =$ ________.

(2) Define : simple pole.

(3) Define : residue.

(4) Define : isolated singular point.

(B) Attempt any one out of two : **2**

(1) Find the pole of $f(z)=\frac{d^{oz}(z+2)}{z^3 2z^2-5z+6}$

(2) Find the residue of $f(z)=\frac{e^{3z}}{z(z-1)} at\ z_0=0$

(C) Attempt any one out of two : **3**

(1) If Z_0 the pole of order m for a complex valued function $f(z)$, then show that

$\operatorname{Re} s(f(z), z_0)=\frac{\varphi^{m-1}(z_0)}{(m-1)!}$, where $\phi(z)=(z-z_0)^m f(z)$

(2) Using residue theorem prove that

$\int_{-\infty}^{\infty} \frac{1}{(1+x^2)^3} dx = \frac{3\pi}{8}$.

(D) Attempt any one out of two : **5**

(1) Evaluate $\int_{|z|=3} \frac{e^{tz}}{z^2(z^2+2z+2)} dz$.

(2) State and prove Cauchy-Residue theorem.